8-2 L'Hopital's Rule

Learning Objectives:

I can use L'Hopital's rule to find the limits of the form $\frac{0}{0}, \frac{\infty}{\infty}$

I can use L'Hopital's rule to find the limits of the form $\infty - \infty, \infty \cdot 0, 1^{\infty}, 0^{0}, \infty^{0}, 0^{\infty}$

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Ex1. Evaluate

1.) $\lim_{x \to -1} \frac{x^2 + 7x + 6}{x^2 + 5x + 4}$

2.) $\lim_{x \to \infty} \frac{2x^3 + 2x}{5x^3 + 1}$

3.) $\lim_{x \to 0} \frac{\sin x}{x}$

Indeterminate Forms

Indeterminate

$$\boxed{\frac{0}{0} \quad \frac{\infty}{\infty}}$$

$$0^{\infty} \quad \infty^{0} \quad 1^{\infty} \quad 0^{0}$$

$$\infty - \infty \quad \infty \cdot 0$$

Not Indeterminate

$$0 \Leftarrow \frac{0}{\infty} \qquad \frac{\infty}{0} \Rightarrow \infty$$

$$\frac{3}{\infty} \Rightarrow 0 \qquad \frac{\infty}{3} \Rightarrow \infty \qquad \frac{5}{0} \Rightarrow \infty \qquad \frac{0}{2} \Rightarrow 0$$

L'Hopital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)}$ takes on one of the indeterminate

forms $\frac{\infty}{\infty}$ or $\frac{0}{0}$, then the limit can be

determined by $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

provided $g'(a) \neq 0$, f'(a) exists, and g'(a) exists.

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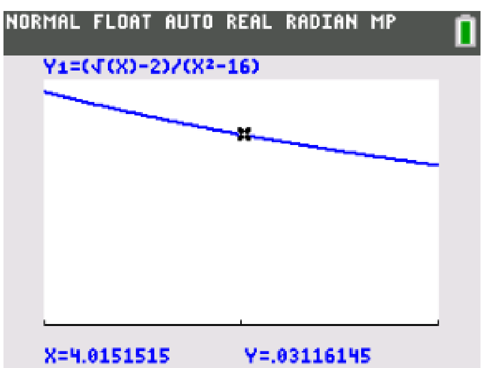
Ex2. Evaluate

1.) $\lim_{x \to 4} \frac{\sqrt{x}-2}{x^2-16}$ $\quad \frac{0}{0}$ ✓ $\quad x^{\frac{1}{2}} - 2$

$$\lim_{x \to 4} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{2x} = \frac{\frac{1}{2}\left(\frac{1}{2}\right)}{8} = \frac{\frac{1}{4}}{8} = \frac{1}{32}$$



NORMAL FLOAT AUTO REAL RADIAN MP

X	Y1				
3.9	.03185				
3.99	.03131				
3.999	.03126				
4	ERROR				
4.001	.03124				
4.01	.03119				
4.1	.03067				

X=

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2.) $\lim_{x\to 0} \dfrac{\sqrt{1+x}-1-\frac{x}{2}}{x^2}$

$\dfrac{1-1-0}{0} = \boxed{\dfrac{0}{0}}$

$\dfrac{(1+x)^{1/2}-1-\frac{x}{2}}{x^2} \Rightarrow \dfrac{\frac{1}{2}(1+x)^{-1/2}-\frac{1}{2}}{2x}$

$\dfrac{\frac{1}{2}-\frac{1}{2}}{2(0)} \Rightarrow \boxed{\dfrac{0}{0}}$

$\dfrac{-\frac{1}{4}(1+x)^{-3/2}}{2} \Rightarrow \dfrac{-\frac{1}{4}}{2} \Rightarrow \boxed{\dfrac{-1}{8}}$

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3.) $\lim_{x \to 0} \frac{\sin x}{x}$

$\frac{\sin 0}{0} = \frac{0}{0}$

$\lim_{x \to 0} \frac{\cos x}{1}$ $\cos(0) = 1$

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4.) $\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$

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$x^{1/2}$

5.) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ $\frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$ LR#1 $\frac{0}{0}$

$= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{2\sqrt{x}}{1} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x}$ $\frac{\infty}{\infty}$ simplify

LR#2

$= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{1} = \frac{0}{1} = \boxed{0}$

$\infty \cdot 0$

6.) $\lim_{x\to\infty} x^2 e^{-x} \Longrightarrow \lim_{x\to\infty} \frac{x^2}{e^x}$

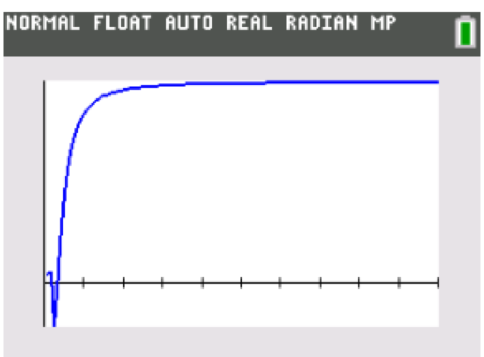
$x^2 \cdot \frac{1}{e^x}$

$\infty \cdot 0$ $\quad x^{-1} \quad -x^{-2}$

7.) $\lim_{x\to\infty} x\cdot\sin\left(\frac{1}{x}\right) \Rightarrow \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$ $\quad \frac{0}{0}$

$$= \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot \cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos(0) = 1$$



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X	Y_1
1	.84147
10	.99833
100	.99998
1000	1

X=

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$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} \qquad \infty - \infty$$

8.) $\lim_{x\to 0}\left(\csc x - \cot x\right)$

$$\lim_{x\to 0} \frac{1-\cos x}{\sin x} \qquad \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sin x}{\cos x} = \frac{0}{1} = \boxed{0}$$

$\frac{1}{0} - \frac{1}{0} \quad \infty - \infty$

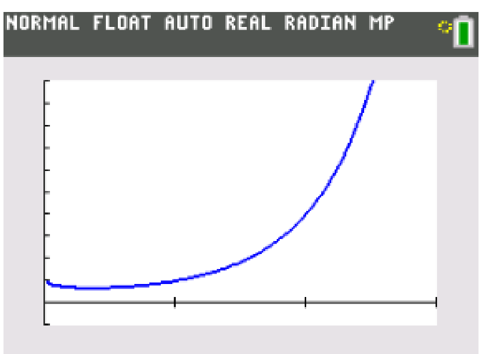
9.) $$\lim_{x\to 1}\left[\frac{x}{x-1}-\frac{1}{\ln x}\right]$$

Ex) $\lim_{x\to 1}\left[\frac{x}{x-1} - \frac{1}{\ln x}\right] \Rightarrow \frac{1}{0} - \frac{1}{0} \Rightarrow \infty - \infty$

$\lim_{x\to 1} \frac{(\ln x)x - (x-1)}{(x-1)\ln x} \Rightarrow \frac{0-0}{0\cdot 0} = \frac{0}{0}$

$\lim_{x\to 1} \frac{(\frac{1}{x})x + \ln x(1) - 1}{(x-1)\frac{1}{x} + \ln x} \Rightarrow \lim_{x\to 1} \frac{1+\ln x - 1}{1 - \frac{1}{x} + \ln x} = \lim_{x\to 1} \frac{1+\ln x - 1}{0+0} = \frac{0}{0}$

$\lim_{x\to 1} \frac{\frac{1}{x}}{\frac{1}{x^2} + \frac{1}{x}} = \frac{1}{1+1} = \frac{1}{2}$



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X	Y_1			
1	1			
.1	.79433			
.01	.95499			
.001	.99312			
1E-4	.99908			

X=1

0^0

10.) $\lim_{x\to 0^+} x^x$

$y = x^x$

$\ln y = \ln x^x$

$\ln y = x \cdot \ln x$

$0 \cdot -\infty$

$\lim_{x\to 0^+} x \cdot \ln x$

$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$ $\frac{-\infty}{\infty}$

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{1}{\cancel{x}} \cdot \frac{-x^{\cancel{2}}}{1}$

$= \lim_{x\to 0^+} (-x) = 0$

$e^0 = \boxed{1}$

11.) $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x \quad 1^\infty$

$\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x \quad 1^\infty \qquad \ln y = x\ln\left(1+\frac{1}{x}\right) \qquad \ln y = \frac{\ln\left(1+\frac{1}{x}\right)}{x^{-1}}$

$\ln y = \frac{\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}}{-x^{-2}} \qquad \ln y = \frac{1}{1+\frac{1}{x}}\cdot\cancel{\frac{-1}{x^2}}\cdot\cancel{\frac{-x^2}{1}}$

$\ln y = \frac{1}{1+\frac{1}{x}} \qquad \ln y = 1 \qquad e^1 = e$

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Homework

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